

Math 6490: Assignment 2

Due Thursday, February 26

1. Penrose wallpaper:
 - (a) Make lots of copies of the attached Penrose tiling. The tiles are all of two shapes: a skinny rhombus, with angles 36° and 144° and a fat rhombus with angles 72° and 108° .
 - (b) By trial and error, connect a subset of the vertices together with red line segments so that we can see a larger Penrose tiling superimposed on top of the smaller one.
 - (c) Continue this pattern throughout the whole page.
 - (d) Find the ratio of the sides of the larger tiles to the sides of the original tiles.
 - (e) Show that each larger tile is subdivided in exactly the same way. Try to find rules for this subdivision.
 - (f) find a Euclidean similarity that maps the tiling by small tiles onto the tiling by large tiles.
2. Read Chapter 2 on complex number geometry.
3. Project 2.1: Find all the Gaussian primes $m + ni$ with $\sqrt{m^2 + n^2} \leq 20$. Make a plot. We will work on this in Maple.

For extra credit, devise a program to factor a Gaussian integer $m + ni$ into prime Gaussian integers.
4. Project 2.2: If $x + iy = (u + iv)^2$, write out formulas for u and v in terms of x and y . Explain how to choose the solution with $u > 0$ or $u = 0$ and $v \geq 0$.
5. Project 2.3: We will devise Maple programs to produce these kinds of pictures. Read the Maple help page on `conformal`.
6. Project 2.4: Explain how every rigid motion can be written in the form $T(z) = Az + B$ or $T(z) = A\bar{z} + B$, where z is a complex variable and A, B are complex constants. What about Euclidean similarities?
7. Project 2.5: We will work on drawing complex spirals in Maple.
8. Project 2.6: Prepare a group law table with T_1, \dots, T_8 running along both the top of the columns and the left side of the rows. Then in the entry corresponding to the row labelled T_i and the column labelled T_j place the result of composition $T_i \circ T_j$. (It will be some T_k .) This will show the structure of the group of these 8 transformations.