

**EXAM 2**  
**MATH 3013 SECTION 002, SPRING 2009**

INSTRUCTOR: WEIPING LI

**Print Name and Student #**

**SHOW WORK FOR CREDIT !!! SHOW WORK FOR CREDIT !!!**

(1) (10pts) Enlarge the given independent set  $\{(1, 0, -1, 1)\}$  in  $R^4$ .

(2) (5pts) Determine whether the given set of vectors  $\{x, x^2 + 1, (x - 1)^2\}$  is a basis for  $P_2$ .

- (3) (10pts) Find the dimension of  $W = \text{span}\{w_1, w_2, w_3, w_4\}$ , where  $w_1 = (1, -3, 1)$ ,  $w_2 = (-2, 6, -2)$ ,  $w_3 = (2, 1, -4)$ ,  $w_4 = (-1, 10, -7)$ .

- (4) (15pts) Find (i) the rank of  $A$  and the nullity of  $A$ , (ii) a basis for the row space, (iii) a basis for the column space of

$$A = \begin{pmatrix} 1 & 3 & 0 & -1 & 2 \\ 0 & -2 & 4 & -2 & 0 \\ 3 & 11 & -4 & -1 & 6 \\ 2 & 5 & 3 & -4 & 0 \end{pmatrix}.$$

(5) (5pts) Find the polynomial in  $P_3$  whose coordinate vector relative to the ordered basis  $B = \{x^3 + x^2, x^2 - x, -x + 1, 1\}$  is  $(-1, 1, 0, 2)$ .

(6) (15pts) Let  $T : P_3 \rightarrow P_3$  be defined by  $T(p(x)) = (x - 1) \cdot D(p(x))$ , where  $D(p(x))$  is the derivative of  $p(x)$ . Let  $B = B' = \{x^3, x^2, x, 1\}$  be the ordered basis. (i) Find the matrix representation  $A$  of  $T$ . (ii) Use  $A$  to compute  $T(x^2 - 2x + 3)$ . (iii) Is  $T$  one-to-one?

(7) (15pts) Given a linear transformation  $T(x_1, x_2, x_3) = (x_1 - 2x_2 + x_3, x_2 - x_3, 2x_2 - 3x_3)$ .

(a) Find its standard matrix representation.

(b) Is the linear transformation one-to-one?

(c) Is the linear transformation invertible ?

(8) (15pts) Let  $T$  be a linear transformation such that  $T(1, 2) = (1, 0, -1)$  and  $T(3, 5) = (1, -1, 1)$ .

(i) Find  $T(x, y)$ .

(ii) Using (i), compute  $T(-1, 1)$ .

(9) (10pts) Find the coordinate vector of the polynomial  $4x^3 - 9x^2 + x$  relative to the ordered basis  $B_1 = \{1, (x - 1), (x - 1)^2, (x - 1)^3\}$ .