

EXAM 1
MATH 3013 SECTION 002, SPRING 2009

INSTRUCTOR: WEIPING LI

Print Name and Student #

SHOW WORK FOR CREDIT !!! SHOW WORK FOR CREDIT !!!

- (1) (5pts) Fill in the missing entries in the 4×4 matrix below so that the matrix is symmetric.

$$\begin{pmatrix} 1 & -1 & & 5 \\ & 4 & & 8 \\ 2 & -7 & -1 & \\ & & 6 & 3 \end{pmatrix}.$$

- (2) (15pts) Let $\mathbf{u} = (1, -1, 0)$ and $\mathbf{v} = (0, 1, -1)$.
- (i) Find the angle between \mathbf{u} and \mathbf{v} ;
 - (ii) find $\|\mathbf{u} + 2\mathbf{v}\|$;
 - (iii) Find the unit vector parallel to $\mathbf{u} + 2\mathbf{v}$, having the opposite direction.

(3) (10pts) Prove that $(2, 0, 4)$, $(4, 1, -1)$ and $(6, 7, 7)$ are vertices of a right triangle in \mathbb{R}^3 .

(4) (15pts) Determine whether the vectors $v_1 = (1, 1, -1)$, $v_2 = (-3, -2, 1)$ and $v_3 = (1, 3, -5)$ form a basis for \mathbf{R}^3 .

(5) (10pts) Let $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}$. Compute (i) $A \cdot B$; and (ii) $B \cdot A$.

- (6) (15pts) (a) Determine whether the matrix $A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 5 & -3 \\ -3 & 2 & -4 \end{pmatrix}$ is invertible, and find its inverse.

- (b) Express the invertible matrix $B = \begin{pmatrix} 2 & 9 \\ 1 & 4 \end{pmatrix}$ as a product of elementary matrices.

(7) (15pts) Find a basis for the solution set of the homogeneous linear system

$$x_1 - x_2 + x_3 - x + x_4 = 0$$

$$x_2 + x_3 = 0$$

$$x_1 + x_2 - x_3 + 3x_4 = 0.$$

(8) (15pts) Solve the given system

$$2x_1 - x_2 + 2x_3 = -3, \quad 2x_1 + x_2 - 2x_4 = 1.$$

Express your solution as a particular solution with solutions from homogeneous system.