

MAXWELL'S EQUATIONS

1 Coulomb's Law

Given a point charge Q at the origin in \mathbb{R}^3 and a point charge q at the point $\mathbf{r} = (x, y, z)$ in \mathbb{R}^3 , Coulomb's Law states that the magnitude of the force \mathbf{F} acting on q is directly proportional to the product Qq of the two charges and inversely proportional to the square r^2 of the distance $r = \|\mathbf{r}\| = \sqrt{x^2 + y^2 + z^2}$ between them. Moreover this force points in the direction \mathbf{r}/r from Q to q . Thus

$$\mathbf{F} = \frac{k_E Q q \mathbf{r}}{r^2} = \frac{k_E Q q \mathbf{r}}{r^3}$$

where k_E is a constant which depends on the system of units chosen. We can break Coulomb's Law into two parts by defining the electric field \mathbf{E} to be

$$\mathbf{E} = \frac{k_E Q \mathbf{r}}{r^3}$$

and then noting that $\mathbf{F} = \mathbf{E}q$.

The electric field of a point charge is a conservative vector field, i.e. it is the gradient of a scalar field. This can be seen from the fact that

$$\nabla \frac{1}{r} = -\frac{\mathbf{r}}{r^3}.$$

Hence

$$\nabla \frac{k_E Q}{r} = k_E Q \nabla \frac{1}{r} = -\frac{k_E Q \mathbf{r}}{r^3} = -\mathbf{E}$$

and so $\mathbf{E} = -\nabla \frac{k_E Q}{r}$. The scalar field $\varphi = \frac{k_E Q}{r}$ is called the *electric potential* of the point charge Q . It has the property that $\mathbf{E} = -\nabla \varphi$.

Recall that $\text{curl grad } g = \mathbf{0}$ for any scalar field g . It follows that

$$\text{curl } \mathbf{E} = \mathbf{0}$$

for the electric field of a point charge.

Recall also that $\text{div grad } g = \nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2}$, i.e. the divergence of the curl is the *Laplacian*. A calculation shows that $\nabla^2(1/r) = 0$. Thus

$$\nabla^2 \varphi = 0.$$

This is called *Laplace's Equation*. Any solution of this equation is called a *harmonic function*. Thus the electric potential of a point charge is a harmonic function.

Since $\mathbf{E} = -\text{grad } \varphi$ we have $\text{div } \mathbf{E} = -\text{div grad } \varphi = -\nabla^2 \varphi$ and thus

$$\text{div } \mathbf{E} = 0$$

for the electric field of a point charge.

2 Gauss' Law

Suppose $\mathbf{S} = (S, \mathbf{n})$ is a sphere of radius a centered at the origin and oriented by the outward normal $\mathbf{n} = (x, y, z)/a$. For a point $\mathbf{r} = (x, y, z)$ on S we have that $r = a$. We compute the *electric flux* across \mathbf{S} :

$$\begin{aligned} \iint_{\mathbf{S}} \mathbf{E} \cdot d\mathbf{S} &= \iint_S \mathbf{E} \cdot \mathbf{n} dS = \iint_S \left(\frac{k_E Q \mathbf{r}}{r^3} \right) \cdot \left(\frac{\mathbf{r}}{r} \right) dS = \\ &= \iint_S \frac{k_E Q \mathbf{r} \cdot \mathbf{r}}{r^4} dS = \iint_S \frac{k_E Q r^2}{r^4} dS = \iint_S \frac{k_E Q}{r^2} dS = \iint_S \frac{k_E Q}{a^2} dS = \\ &= \frac{k_E Q}{a^2} \text{Area}(S) = \frac{k_E Q}{a^2} 4\pi a^2 = 4\pi k_E Q \end{aligned}$$

Note that this result does not depend on the radius a of the sphere. In fact, it does not depend on the fact that Q is at the center of the sphere or even on the fact that our surface is a sphere. This surprising fact is a consequence of Gauss' Divergence Theorem, which we can see as follows.

Let W be a solid region in \mathbb{R}^3 such that Q is in the interior of W . This means that there is a small number $a > 0$ such that the open ball $B = \{(x, y, z) \mid x^2 + y^2 + z^2 < a^2\}$ of radius a with center the origin is contained in the interior of W . If we remove B from W we get a new solid region $W' = W - B$ such that $\partial W'$ consists of two pieces: ∂W and the sphere $S = \{(x, y, z) \mid x^2 + y^2 + z^2 = a^2\}$.

We apply the Divergence Theorem to W' . This says that

$$\iint_{\partial W'} \mathbf{E} \cdot d\mathbf{S} = \iiint_{W'} \text{div } \mathbf{E} dV$$

where $\partial W'$ is oriented by the outward pointing normal.

Now on the ∂W part of $\partial W'$ the outward normal is the same as the one for ∂W . On the spherical part S of $\partial W'$ the normal that points *outward* from W' is the normal that points *inward* from S towards the origin. This is the *negative* of the normal that we used above to compute the electric flux across \mathbf{S} . Recalling that $-\mathbf{S} = (S, -\mathbf{n})$ and that vector surface integrals change sign when you change orientation we have that

$$\iint_{\partial W'} \mathbf{E} \cdot d\mathbf{S} = \iint_{\partial W} \mathbf{E} \cdot d\mathbf{S} - \iint_{\mathbf{S}} \mathbf{E} \cdot d\mathbf{S}$$

and thus

$$\iint_{\partial W} \mathbf{E} \cdot d\mathbf{S} - \iint_{\mathbf{S}} \mathbf{E} \cdot d\mathbf{S} = \iiint_{W'} \text{div } \mathbf{E} dV$$

Now recall that in the previous section we showed that $\operatorname{div} \mathbf{E} = 0$, so the integral on the right is zero, and we have

$$\iint_{\partial W} \mathbf{E} \cdot d\mathbf{S} = \iiint_W \operatorname{div} \mathbf{E} \, dV = 0$$

Next suppose that we have several charges Q_1, \dots, Q_n inside W . Each Q_i contributes a part \mathbf{E}_i of the total electric field $\mathbf{E} = \mathbf{E}_1 + \dots + \mathbf{E}_n$, and thus a part $4\pi k_E Q_i$ of the total electric flux $4\pi k_E Q$, where the total charge inside W is $Q = Q_1 + \dots + Q_n$. This gives us the general form of Gauss' Law for finite sets of point charges:

$$\iint_{\partial W} \mathbf{E} \cdot d\mathbf{S} = 4\pi k_E Q$$

3 Continuous charge distributions

If the number of particles under consideration is very large, then it is difficult to apply Coulomb's Law to the situation. Instead, we pretend that the charge is specified by a continuous *charge density* ρ . This is a continuous scalar field $\rho : \mathbb{R}^3 \rightarrow \mathbb{R}$, measured in, for example, coulombs per cubic meter, such that the total charge in any solid region W is given by

$$Q = \iiint_W \rho \, dV$$

Since we no longer have point charges we no longer have Coulomb's Law, and so we need some other way to try to find the electric field \mathbf{E} . We start by replacing Coulomb's Law by Gauss' Law.

$$\iint_{\partial W} \mathbf{E} \cdot d\mathbf{S} = 4\pi k_E Q$$

for every solid region W . We next replace the left hand side of this equation by using Gauss' Divergence Theorem and the right hand side by using the expression given above for Q to get

$$\iiint_W \operatorname{div} \mathbf{E} \, dV = \iiint_W 4\pi k_E \rho \, dV$$

Since this must be true for every region W one can show that the two functions being integrated must be equal. So we have

$$\operatorname{div} \mathbf{E} = 4\pi k_E \rho$$

Note that this is different from the result $\operatorname{div} \mathbf{E} = 0$ that we obtained for point charges. A different model gives different equations.

The constant $4\pi k_E$ is rather cumbersome; let's rename it $1/\epsilon_0$. Then we have

$$\boxed{\operatorname{div} \mathbf{E} = \rho/\varepsilon_0}$$

This is the first of Maxwell's four equations, the differential form of Gauss' Law. Physically, this says that near a point at which the charge density is positive the electric field should more or less be pointing away from the point.

What about curl \mathbf{E} ? For the moment let's assume that $\operatorname{curl} \mathbf{E} = \mathbf{0}$, as it was in the point charge case. (Later on this will change.)

4 The Biot-Savart Law

We now turn to the study of the magnetic field \mathbf{B} . Both the field \mathbf{B} and the history of its study are more complicated than the field \mathbf{E} and the history of its study. We will ignore the real history with its twists and turns and present a kind of "pseudo-history" that gives the development for \mathbf{B} a structure similar to that for \mathbf{E} .

As in the case of \mathbf{E} we will break things into two parts: a formula for the force \mathbf{F} that \mathbf{B} exerts on a charge q and a formula for \mathbf{B} .

If you put a point charge q in a magnetic field \mathbf{B} and the charge is initially at rest, then nothing will happen. \mathbf{B} exerts no force on q . But if q is moving with velocity \mathbf{v} , then \mathbf{B} exerts a force on q which is perpendicular to both \mathbf{B} and \mathbf{v} . More precisely

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

If you throw q into a constant magnetic field \mathbf{B} with \mathbf{v} perpendicular to \mathbf{B} , then q will move in a circle.

So the magnetic field acts on *moving* charges. It is created by moving charges as well, i.e by *electric currents*. Suppose we have a current I which flows upward along an infinitely long and infinitely thin wire which stretches along the z -axis. We will measure I in *amperes*, i.e coulombs per second. Then the magnetic field is given by the following formula, which is a version of the Biot-Savart Law.

$$\mathbf{B} = \frac{k_B I(-y, x, 0)}{x^2 + y^2}$$

The constant k_B depends on the units used. This will be our analog of Coulomb's Law. Instead of dealing with point charges we are dealing with "line currents" as the source of the magnetic field. Note that the formula says that \mathbf{B} is horizontal, lying in a plane perpendicular to the wire, and curls around the z -axis.

As an exercise, you can show that for this formula $\operatorname{div} \mathbf{B} = 0$ and $\operatorname{curl} \mathbf{B} = \mathbf{0}$.

Despite the fact that $\operatorname{curl} \mathbf{B} = \mathbf{0}$ it turns out that \mathbf{B} , unlike \mathbf{E} , is NOT a gradient vector field, as we will see below.

5 Ampère's Law

Now let $\mathbf{C} = \{(x, y, z) \mid x^2 + y^2 = b^2, z = 0\}$, the circle of radius b in the x - y plane, oriented in the counter-clockwise direction. We compute the line integral of \mathbf{B} around this curve using the parametrization $x = b \cos \theta$, $y = b \sin \theta$, where $0 \leq \theta \leq 2\pi$. We have $dx = -b \sin \theta d\theta$ and $dy = b \cos \theta d\theta$. This gives

$$\int_{\mathbf{C}} \mathbf{B} \cdot d\mathbf{s} = k_B I \int_{\mathbf{C}} \frac{-y dx + x dy}{x^2 + y^2} =$$

$$k_B I \int_0^{2\pi} \frac{(-b \sin \theta)(-b \sin \theta d\theta) + (b \cos \theta)(b \cos \theta d\theta)}{(b \cos \theta)^2 + (b \sin \theta)^2} = k_B I \int_0^{2\pi} 1 d\theta = 2\pi k_B I$$

Note that this does not depend on the radius b of the circle. It also does not depend on the fact that the circle lies in the x - y plane; any horizontal plane will give the same answer. In fact we will see shortly that it does not depend on the fact that we are integrating over a circle.

Suppose we have a surface S whose boundary ∂S is a single simple closed curve. Suppose also that the z -axis hits S in exactly one point (and that it actually pierces through S rather than being tangent to it). We will further assume that S contains an open disk D of radius a which lies in a horizontal plane and meets the z axis in its center. We make S an oriented surface \mathbf{S} by giving it the normal vector field \mathbf{n} whose value at the intersection of S with the z -axis is $(0, 0, 1)$.

We now let $S' = S - D$. This is a surface whose boundary has two pieces: ∂S and C . The orientation of \mathbf{S} gives ∂S the same orientation as before but gives C the opposite orientation. More concisely, $\partial \mathbf{S}' = \partial \mathbf{S} - C$. So,

$$\int_{\partial \mathbf{S}'} \mathbf{B} \cdot d\mathbf{s} = \int_{\partial \mathbf{S}} \mathbf{B} \cdot d\mathbf{s} - \int_C \mathbf{B} \cdot d\mathbf{s}$$

We now apply Stokes' Theorem and the fact that $\text{curl } \mathbf{B} = \mathbf{0}$ to get

$$\int_{\partial \mathbf{S}'} \mathbf{B} \cdot d\mathbf{s} = \iint_{\mathbf{S}'} (\text{curl } \mathbf{B}) \cdot d\mathbf{S} = 0$$

This implies that

$$\int_{\partial \mathbf{S}'} \mathbf{B} \cdot d\mathbf{s} = \int_C \mathbf{B} \cdot d\mathbf{s}$$

and so we have

$$\int_{\partial \mathbf{S}'} \mathbf{B} \cdot d\mathbf{s} = 2\pi k_B I$$

This is called *Ampère's Law*.

6 Continuous Current Distributions

We now change models, replacing “line currents” by a continuous distribution of current. We do this by supposing that there is a vector field \mathbf{J} , the *current density*, with the property that for any oriented surface \mathbf{S} the current I flowing across \mathbf{S} in the direction specified by the orientation is given by

$$I = \iint_{\mathbf{S}} \mathbf{J} \cdot d\mathbf{S}$$

Note that \mathbf{J} has the unit of amperes per square meter.

Since we no longer have line currents we no longer have the Biot-Savart Law, so we need a substitute. We replace the Biot-Savart Law by Ampère’s Law.

$$\int_{\partial\mathbf{S}'} \mathbf{B} \cdot d\mathbf{s} = 2\pi k_B I$$

We apply Stokes’ Theorem to the left hand side of this equation and our formula for I in terms of \mathbf{J} to the right hand side to get

$$\iint_{\mathbf{S}} \text{curl } \mathbf{B} \cdot d\mathbf{S} = \iint_{\mathbf{S}} 2\pi k_B \mathbf{J} \cdot d\mathbf{S}$$

Since this must hold for all oriented surfaces \mathbf{S} the two functions being integrated must be equal, so we have

$$\text{curl } \mathbf{B} = 2\pi k_B \mathbf{J}$$

We simplify the notation by setting $\mu_0 = 2\pi k_B$. This gives

$$\text{curl } \mathbf{B} = \mu_0 \mathbf{J}$$

This is the differential form of Ampère’s Law. It is almost one of Maxwell’s equations. (There is a surprise later on.)

What about $\text{div } \mathbf{B}$? If it were non-zero at a point, then we would expect \mathbf{B} to point away from the point when we are very close to it. This would be a *magnetic monopole*. Such things have never been observed, so we will keep

$$\boxed{\text{div } \mathbf{B} = 0}$$

as one of Maxwell’s equations.

7 Faraday's Law

Suppose you take an oriented surface \mathbf{S} with boundary $\partial\mathbf{S}$ a single simple closed curve. Hold it in a magnetic field \mathbf{B} . The quantity

$$\iint_{\mathbf{S}} \mathbf{B} \cdot d\mathbf{S}$$

is called the *magnetic flux* across the surface. Now suppose that you move the surface in such a way that this flux changes with time. If you think of $\partial\mathbf{S}$ as a wire, then a current will start to flow in the wire. This current is propelled by an electric field \mathbf{E} that appears in the wire. This experimental phenomenon is described by the equation

$$\int_{\partial\mathbf{S}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint_{\mathbf{S}} \mathbf{B} \cdot d\mathbf{S}$$

This is called *Faraday's Law*. Electric generators are based on it; you use a turbine powered by water, steam, gas, etc. to rotate what amounts to big loops of wire in a magnetic field.

As usual we can turn this into a differential version. We apply Stokes' Theorem to the left hand side of the equation and move the $-d/dt$ inside the integral sign, where it becomes a partial derivative $-\partial/\partial t$.

$$\iint_{\mathbf{S}} (\text{curl } \mathbf{E}) \cdot d\mathbf{S} = \iint_{\mathbf{S}} -\frac{\partial\mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

Since this must hold for all oriented surfaces the two functions being integrated must be equal, so we have the differential form of Faraday's Law

$$\boxed{\text{curl } \mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}}$$

This is another of Maxwell's four equations. Note that it replaces the equation $\text{curl } \mathbf{E} = \mathbf{0}$ that we had earlier.

8 Why Electricity Doesn't Exist and How Maxwell Fixed That

Let's summarize what we have so far.

- (1) Gauss' Law: $\text{div } \mathbf{E} = \rho/\epsilon_0$
- (2) No Magnetic Monopoles: $\text{div } \mathbf{B} = 0$

(3) Faraday's Law: $\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

(4) Ampère's Law: $\text{curl } \mathbf{B} = \mu_0 \mathbf{J}$

These laws were known in some form or other before Maxwell. He looked at them and realized that there was a problem. There is another important law we need to add to these, the Law of Charge Conservation. Suppose you have some electric charge in a region W and the charge starts moving out of W . You don't expect it to just suddenly disappear. You expect it to cross ∂W to get outside. The way the charge Q decreases is by a current I flowing outward across ∂W . In other words

$$\frac{dQ}{dt} = -I$$

Expressing these in terms of integrals we get

$$\frac{d}{dt} \iiint_W \rho dV = - \iint_{\partial W} \mathbf{J} \cdot d\mathbf{S}$$

We move the d/dt inside the integral on the left (as $\partial/\partial t$) and apply the Divergence Theorem to the integral on the right to get

$$\iiint_W \frac{\partial \rho}{\partial t} dV = \iiint_W -\text{div } \mathbf{J} dV$$

In the usual way this implies that

$$\boxed{\text{div } \mathbf{J} = -\frac{\partial \rho}{\partial t}}$$

This is called the *Continuity Equation*.

So, what's the problem? Recall that $\text{div curl} = \mathbf{0}$. Using Ampère's Law and the Continuity Equation we get

$$0 = \text{div curl } \mathbf{B} = \text{div } \mu_0 \mathbf{J} = \mu_0 \text{div } \mathbf{J} = -\mu_0 \frac{\partial \rho}{\partial t}$$

What this says is that a charge distribution cannot change with time. In particular charge cannot move from one place to another. So electricity doesn't exist.

Well, something is wrong here. Maxwell suspected that Ampère's Law was the culprit. He tried to fix it by adding an extra term to it so that we wouldn't get $\partial \rho / \partial t = 0$. Let's call this extra term \mathbf{X} . We have

$$\text{curl } \mathbf{B} = \mu_0 \mathbf{J} + \mathbf{X}$$

We take the divergence of both sides.

$$0 = \operatorname{div} \operatorname{curl} \mathbf{B} = \mu_0 \operatorname{div} \mathbf{J} + \operatorname{div} \mathbf{X} = -\mu_0 \frac{\partial \rho}{\partial t} + \operatorname{div} \mathbf{X}$$

So we need to choose \mathbf{X} so that $\operatorname{div} \mathbf{X} = \mu_0 \frac{\partial \rho}{\partial t}$. How do we find such an \mathbf{X} ? Look at the only one of the equations (1)-(4) that contains ρ , namely Gauss' Law $\operatorname{div} \mathbf{E} = \rho/\varepsilon_0$. Rewrite it as

$$\rho = \varepsilon_0 \operatorname{div} \mathbf{E} = \operatorname{div} \varepsilon_0 \mathbf{E}$$

Differentiate both sides with respect to t to get

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t} \operatorname{div} \varepsilon_0 \mathbf{E} = \operatorname{div} \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Finally multiply both sides by μ_0 to get

$$\mu_0 \frac{\partial \rho}{\partial t} = \mu_0 \operatorname{div} \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \operatorname{div} \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Hence we can choose $\mathbf{X} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ and thus get

$$\operatorname{curl} \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

This is the *Ampère-Maxwell Law*.

Note that the extra term Maxwell added to Ampère's Law makes it look somewhat like Faraday's Law. In the same way that Faraday's Law says that a changing magnetic flux creates an electric field this suggests that a changing electric flux should create a magnetic field. But it turns out that the coefficient $\mu_0 \varepsilon_0$ is very small, so this would be a very small effect. There was no evidence to suggest its existence before Maxwell published his results in 1865. A direct experimental measurement of this phenomenon was not obtained until the 1920's. However, as we will see, there is an astonishing consequence of it which Maxwell predicted, which was experimentally verified a couple of decades later, and caused a vast technological revolution.

9 Maxwell's Equations

Putting all this together we have the four Maxwell's Equations.

$$\begin{aligned} \operatorname{div} \mathbf{E} &= \rho/\varepsilon_0 \\ \operatorname{div} \mathbf{B} &= \mathbf{0} \\ \operatorname{curl} \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \operatorname{curl} \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{aligned}$$

10 Electromagnetic Waves

Suppose we now consider the case of Maxwell's Equations in a vacuum, so $\rho = 0$ and $\mathbf{J} = \mathbf{0}$.

$$\begin{aligned} \operatorname{div} \mathbf{E} &= 0 \\ \operatorname{div} \mathbf{B} &= 0 \\ \operatorname{curl} \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \operatorname{curl} \mathbf{B} &= \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{aligned}$$

Notice that they look a bit more symmetrical. Let's play around with them. There is a vector calculus identity which relates all the major derivatives we have studied. For any vector field \mathbf{F} we have

$$\operatorname{grad} \operatorname{div} \mathbf{F} = \operatorname{curl} \operatorname{curl} \mathbf{F} + \nabla^2 \mathbf{F}$$

Recall that the last term on the right is the vector Laplacian ($\nabla^2 F_1, \nabla^2 F_2, \nabla^2 F_3$) where $\mathbf{F} = (F_1, F_2, F_3)$.

Applying this identity to \mathbf{E} we get

$$\operatorname{grad} \operatorname{div} \mathbf{E} = \operatorname{curl} \operatorname{curl} \mathbf{E} + \nabla^2 \mathbf{E}$$

Since $\operatorname{div} \mathbf{E} = 0$ and $\operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ we get

$$\mathbf{0} = -\operatorname{curl} \frac{\partial \mathbf{B}}{\partial t} + \nabla^2 \mathbf{E}$$

so we have

$$\nabla^2 \mathbf{E} = \operatorname{curl} \frac{\partial \mathbf{B}}{\partial t} = \frac{\partial}{\partial t} \operatorname{curl} \mathbf{B}$$

Since $\operatorname{curl} \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ we get

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

A similar computation gives the equation

$$\nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Recall that the *wave equation* has the form

$$\nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

and describes a wave (water wave, sound wave, etc.) which propagates with velocity c . So we have vector versions of the wave equation for the electric and magnetic fields. This suggested to Maxwell the possibility that electromagnetic waves exist. A changing magnetic field creates via Faraday's Law a changing electric field which by Maxwell's addition to Ampère's Law creates a changing magnetic field and so on. As one field decreases the other increases and vice versa, thereby creating a wave which propagates through space with the velocity $c = 1/\sqrt{\mu_0 \epsilon_0}$.

In 1886 Hertz experimentally created and detected such waves, now known as radio waves. These have had some practical applications over the past century.

What is the speed c ? Maxwell computed this with the figures available for μ_0 and ϵ_0 in his day and came up with

$$c = 310,740,000 \text{ meters/sec}$$

He noticed that this was close to the number

$$v = 298,000,000 \text{ meters/sec}$$

which had been obtained by Foucault in 1860 for the speed of light. He concluded that the difference was due to experimental error and that light was an electromagnetic wave.