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TO THE STUDENT

1. Introduction:

Waterhammer is a popular term for unsteady flow within a pipe or piping system. People usually are familiar with waterhammer as an occasional clangor from the water pipes in their home or apartment complex. In such a setting, one may not have considered the effects of waterhammer as being a serious problem for industries such as Conoco. However, with larger equipment, long distances, and liquid flowing faster and faster, the effects of waterhammer can become disastrous. Since Conoco pumps oil and coal slurry through large pipeline systems, high pressures and vibrational damage can destroy certain materials and disrupt normal circuit control. For these reasons, a study of waterhammer and its effects is important.

In this problem, you shall consider a simple system in which the closing of a valve causes a pressure wave to travel through water at the speed of sound. It is this pressure wave that is commonly referred to as waterhammer. In studying this wave, a technique of analysis might be to design and build a particular system and test various value closures experimentally. This is obviously expensive, but more importantly, it does not give the engineer much knowledge about the nature of the pressure wave. Thus he is powerless to design a system which incorporates control parameters in advance.

Engineers have known for a long time empirical methods for resolving waterhammer. One method to solve a waterhammer problem is to make the pipes bigger so as to reduce the velocity of the fluid without reducing the total flow. But how much bigger is big enough? Excess size can be severely expensive, while a single shut-down due to a deficient size can cost millions of dollars per month. A second method of reducing waterhammer is to install tanks (called surge tanks) to hold excess flow when the fluid experiences excessive pressures. Surge tanks cost money, and tend to reduce fluid momentum. Unnecessary reductions in momentum can be extremely costly. These empirical methods do not lend themselves to preventive design.

With the advent of modern digital computers, it is possible to simulate the pressure wave in a pipe. Such a computer simulation is a crucial step in analyzing waterhammer in a relatively inexpensive way. This problen is basically to provide a computer simulation for waterhammer in a single system.

2. Fluid Flow in a Pipe:

The specific problem concerns the flow of water through a horizontal pipe from a reservoir to a hydro-electric plant and the effects of fluid transients or surges known as waterhammer. In this pipeline, there are times when the water must flow through the turbines and times when it must not. This is controlled by a valve. The surges are produced when this long column of rapidly moving water is stopped quickly by this valve. The fluid molecules tend to pack up at the valve during closure as the size of in the direction of flow or stream

the open area at the valve decreases. If we consider the case of instant ~~closure, then,~~ since water is incompressible, the pipe near the valve expands microscopically to accommodate the packing, and the pressure at the valve rises. The fluid upstream continues to move downstream, causing layer upon layer of fluid molecules to pack up. The high pressure moves upstream as a wave, bringing the fluid to rest as it passes, compressing it, and expanding the pipe. When the wave reaches the upstream end of the pipe, all the fluid is under pressure.

The unbalanced forces at the upstream end (the reservoir) cause an acceleration which in this case is a change in the direction of velocity, and the fluid begins to flow in the reverse direction, beginning at the reservoir. This flow returns the pressure to the valve which was normal before closure, and the pipe walls return to normal. The process travels downstream toward the valve at the speed of sound. Since there is no fluid to support the flow at the valve, a low pressure develops and brings the fluid to rest at that end. This low-pressure wave travels upstream at the speed of sound, bringing the fluid to rest, causes it to expand and allows the pipe walls to contract.

At the instant the low-pressure wave reaches the reservoir, the fluid is at rest, but with lower pressure in the pipe. This unbalanced condition at the reservoir causes water to flow back into the pipe. When the flow reaches the valve, the conditions are the same as those at the start, and the process repeats itself.

Without friction, the process would continue indefinitely. Friction acts to reduce the energy of the system to zero. Even with friction, it is possible to get 15 to 20 rebounds of the shock wave from one closure.

These changes in pressure at the valve are what causes the damage to the pipes and valves.

3. A Mathematical Formulation:

The first step in constructing a computer simulation for waterhammer is to develop a mathematical model or formulation for the problem. Since the derivation of the mathematical model is somewhat detailed, this section is devoted to helping you understand how the mathematical model evolves from physical principles. For a more detailed treatment, you may wish to consult a fluid mechanics text such as Fluid Mechanics, by V. Streeter, McGraw-Hill, 5th edition.

There are two equations that are used to describe the flow. They are called the continuity equation and the equation of motion. The independent variables are chosen to be the distance, x, along the pipe, measured positively downstream, and the time, t. The dependent variables chosen are the so-called piezometric head, H, and the discharge, Q. The piezometric head is the ratio of the pressure to the weight per unit volume of the liquid. Other dependent variables could be used, e.g., pressure and velocity. While it is entirely a matter of choice, the use of the piezometric head permits some simplification. One may elect to use velocity rather than discharge.

Continuity Equation

The continuity equation states that the net mass inflow into a segment of the pipe per unit time must equal the time rate of increase of mass within that segment. Consider a horizontal pipe (illustrated in Figure 1) and a short segment of fluid, Δx. Let A denote the cross-sectional area of the pipe, ρ the mass density (1.94 slugs/ft^3 for water), and Q the discharge in ft^3/sec. For the segment of fluid under consideration, the mass involved is the density times the volume, i.e., $\rho A \Delta x$.

rate of change of mass per unit time in x direction

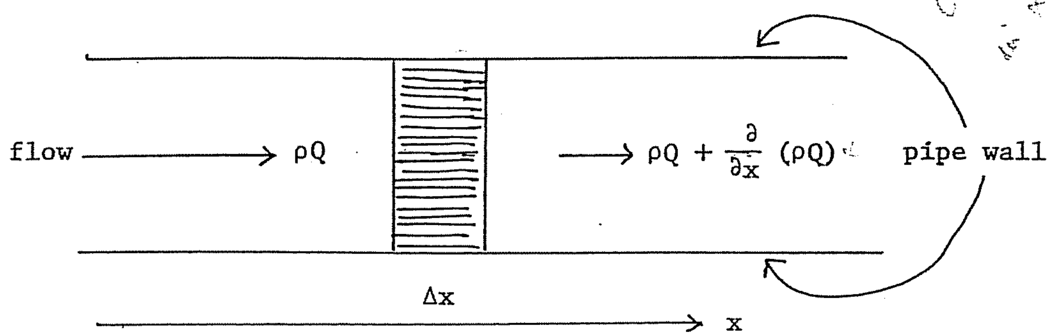

Figure 1

Thus, the time rate of change in mass per unit time is $\partial(\rho A \Delta x)/\partial t$. Now the change in mass inflow may be thought of in simplistic terms as

$$(\text{mass in} - \text{mass out})/\text{unit time} = -\frac{\partial(\rho Q)}{\partial x}\,\Delta x.$$

Thus we have

$$(1) \qquad -\frac{\partial}{\partial x}(\rho Q)\Delta x = \frac{\partial}{\partial t}(\rho A \Delta x).$$

In this approach, we have ignored effects due to possible elongation or shortening of the pipe during stressing, which are not significant here. By expanding Eq. (1) and dividing by $\rho A \Delta x$, we have

$$(2) \qquad \frac{Q}{\rho A}\frac{\partial \rho}{\partial x} + \frac{1}{A}\frac{\partial Q}{\partial x} + \frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial A}{\partial t} = 0.$$

The first term in Eq. (2) is small compared to the third term, and can be neglected. Thus, Eq. (2) reduces to

$$(3) \qquad \frac{1}{A}\frac{\partial Q}{\partial x} + \frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial A}{\partial t} = 0.$$

The relationship between the pressure P and the density ρ is called the equation of state. In this instance, it is given by

$$\text{(4)} \qquad \frac{1}{\rho}\frac{\partial \rho}{\partial t} = \frac{1}{K}\frac{\partial P}{\partial t}$$

The constant K is known as the bulk modulus of elasticity and represents the compressibility of the fluid. Specifically, the bulk modulus is the ratio of the change in unit pressure to the corresponding volume change per unit of volume. For water, $K \doteq 300{,}000 \text{ lb/in}^2$.

The rate of increase of pipe area H with respect to pressure P is given by

$$\text{(5)} \qquad \frac{1}{A}\frac{\partial A}{\partial t} = \frac{D}{eE}\frac{\partial P}{\partial t},$$

where D is the inside diameter of the pipe, e is the wall thickness, and E is Young's modulus. For steel, $E \doteq 30 \times 10^6 \text{ lbs/in}^2$. Substituting Eqs. (4) and (5) into Eq. (3) yields

$$\frac{1}{A}\frac{\partial Q}{\partial x} + \frac{1}{K}\frac{\partial P}{\partial t} + \frac{D}{eE}\frac{\partial P}{\partial t} = 0,$$

or,

$$\text{(6)} \qquad \frac{1}{A}\frac{\partial Q}{\partial x} + \frac{1}{K}\left[1 + \frac{KD}{eE}\right]\frac{\partial P}{\partial t} = 0.$$

It can be shown that the speed of the shock wave, a, is related to the constants in Eq. (6) by

$$a^2 = \frac{K/\rho}{1 + \frac{KD}{eE}}.$$

$H = \frac{P}{mg/V}$

Upon multiplying by A, Eq. (6) becomes

$$\text{(7)} \qquad \frac{\partial Q}{\partial x} + \frac{A}{\rho a^2}\frac{\partial P}{\partial t} = 0.$$

To convert pressure P to piezometric head H, we have

$$\text{(8)} \qquad P = \rho g H,$$

Why?. defn of head

where g, (32.16 ft/sec^2), is the gravitational constant (i.e., ρg is the weight per unit volume of liquid). Although ρ varies with t, it changes little when compared with the change in H with respect to t. Thus,

$$\frac{\partial P}{\partial t} = \rho g \frac{\partial H}{\partial t}.$$

Substituting in Eq. (7), the equation of continuity becomes

$$(9) \qquad \frac{\partial Q}{\partial x} + \frac{Ag}{a^2}\frac{\partial H}{\partial t} = 0.$$

It is this form of the equation of continuity that we shall use in this problem.

The Equation of Motion

The equation of motion is obtained by applying Newton's Second Law of Motion to a small segment of liquid (see Figure 2). We choose a horizontal pipe (for simplicity) and consider a short segment of fluid, Δx, as illustrated.

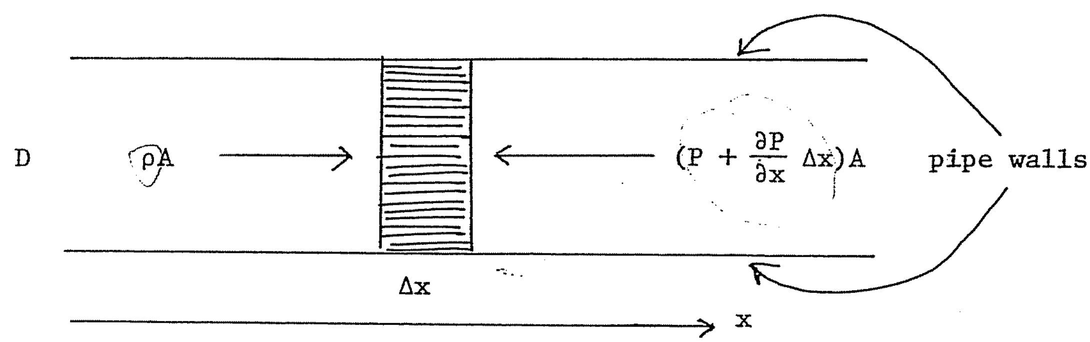

Figure 2

The force pushing the liquid is PA, while the resisting force is approximately $(P + \frac{\partial P}{\partial x}\Delta x)A$. The net force is given by the algebraic sum of these forces and the force due to the walls of the pipe, namely,

$$PA - (P + \frac{\partial P}{\partial x}\Delta x)A - \tau_0 \pi D \Delta x$$

or

$$-\frac{\partial P}{\partial x}\Delta x A - \tau_0 \pi D \Delta x.$$

The quantity τ_0 is the wall stress, i.e., the resistive force per unit area.

This net force is equal to the mass times acceleration, i.e.,

(10) $$-\frac{\partial P}{\partial x}\Delta xA - \tau_0 \pi D\Delta x = \rho A\Delta x \frac{d}{dt}(\text{velocity}).$$

The velocity of the liquid is the ratio of the discharge, Q, to the cross-sectional area, i.e., Q/A. Thus, Eq. (10) becomes

$$-\frac{\partial P}{\partial x}\Delta xA - \tau_0 \pi D\Delta x = \rho A\Delta x \frac{d}{dt}\left(\frac{Q}{A}\right).$$

or

(11) $$\frac{\partial P}{\partial x}A + \tau_0 \pi D + A\rho\frac{d}{dt}\left(\frac{Q}{A}\right) = 0.$$

Recall that $A = \pi(D/2)^2$, and hence $\pi D = \frac{4A}{D}$. Thus Eq. (11) yields

$$\frac{\partial P}{\partial x}A + \tau_0 \frac{4A}{D} + A\rho\frac{d}{dt}\left(\frac{Q}{A}\right) = 0,$$

or

(12) $$\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{4\tau_0}{\rho D} + \frac{d}{dt}\left(\frac{Q}{A}\right) = 0.$$

With respect to time, the cross-sectional area A changes little, and it follows that $\frac{d}{dt}(Q/A) \doteq \frac{1}{A}\frac{dQ}{dt}$. Furthermore, it may be assumed that $\frac{dQ}{dt} \doteq \frac{\partial Q}{\partial t}$. Substituting in Eq. (12) and recalling Eq. (8), it follows that

$$\frac{1}{\rho}\left(\rho g \frac{\partial H}{\partial x}\right) + \frac{4\tau_0}{\rho D} + \frac{1}{A}\frac{\partial Q}{\partial t} = 0,$$

or

(13) $$g\frac{\partial H}{\partial x} + \frac{4\tau_0}{\rho D} + \frac{1}{A}\frac{\partial Q}{\partial t} = 0.$$

The wall stress τ_0 can be expressed in terms of the average velocity, v, by means of the Darcy-Weisback Equation. This equation roughly states that the head loss due to friction in a pipe is "almost" proportional to v^2, L (the length of the pipe), and 1/D, i.e.,

$$\Delta H = f\frac{v^2}{2g}\frac{L}{D}$$

The use of the constant 2g is somewhat artificial. Hydraulic engineers prefer to think in terms of so-called "kinetic head", that is, $v^2/2g$, rather than simply v^2. So the proportionality "constant", f, is adjusted by the factor 2g. Now f turns out to be not quite constant and depends on many things, including the nature of the flow, the roughness of the pipe, the inside diameter, and other pipe characteristics. This friction factor may be determined by using graphs known as Moody Diagrams. In this problem, the friction factor, f, will be given.

In order to determine τ_0, we consider the force associated with the head loss caused by the force $\tau_0(\pi DL)$. The volume of this column of water associated with the head loss is $\Delta H\ A = \left[f\ \frac{v^2}{2g}\frac{L}{D} \right] A$ and the weight of this column of water is $\rho g \left[f\ \frac{v^2}{2g}\frac{L}{D} \right] A$. Since this weight is equivalent to the force acting to push the fluid and overcome the wall-friction, we have

$$\rho g \left[f\ \frac{v^2}{2g}\frac{L}{D} \right] A = \pi D L \tau_0 \ .$$

Recalling that $A = \frac{\pi D^2}{4}$, it follows that

$$\tau_0 = \frac{\rho f v^2}{8} \ .$$

Substituting in Eq. (13) yields

$$g\,\frac{\partial H}{\partial x} + \frac{f v^2}{2D} + \frac{1}{A}\,\frac{\partial Q}{\partial t} = 0,$$

or

$$g\,\frac{\partial H}{\partial x} + \frac{1}{A}\frac{\partial Q}{\partial t} + \frac{f}{2}\ \frac{Q\,|Q|}{DA^2} = 0. \tag{14}$$

The use of the absolute value takes into account the direction of the flow. Thus, the reverse flow may be handled without changing the equation. Eq. (14) is an acceptable form of the equation of motion for this problem.

It will be necessary to find relationships between the head and the velocity, both at the reservoir and at the valve. At the valve, you are to assume that the discharge is proportional to the effective area of the opening A_1 and the square root of the head at that point. For the reservoir the boundary conditions are not as easily stated. For simplicity, one could simply assume a constant head. However, the direction of flow actually determines how the head is computed. If the velocity is zero or negative (reverse flow), the head at the reservoir is given by

$$H_0 + \frac{f L v_0^2}{2gD} + \frac{v_0^2}{2g}\ ,$$

where H_0 is the initial head (before closing) at the valve, f is the friction factor, and v_0 is the velocity of the fluid before closing. If the flow is positive (toward the valve), the head at the reservoir is given by

$$H_0 + \frac{fLv_0^{\,2}}{2gD} + \frac{v_0^{\,2}}{2g} - \frac{v^2}{2g},$$

where v denotes the velocity at that point.

4. <u>The Problem</u>:

Consider a horizontal steel pipe of length 27,000 feet and inside diameter 10 feet which conducts water from a reservoir to a hydo-electric plant. The velocity of the water in this pipe is 10.45 ft/sec. The friction factor, f, is known to be 0.01058. We wish to insert a valve at the end of the pipe. There are many types of valves from which to choose. The rate at which the valve closes is an important factor in determining the character of the water hammer caused by the closing. Basically the problem is to simulate the effects of waterhammer in the pipe resulting from different rates of closure. It is known that the speed of the shock wave is 4671.0 ft/sec.

Some valves are designed to close more rapidly in the initial stages and slower later; others close slowly at first and more rapidly later. Engineers are usually given these closure rates as functions of time. For example, consider a sphincter valve which opens and closes ~~in such a way as~~ to always have a circular orifice for the water to flow through until the valve is completely shut. Let $A_1(t)$ represent the area of the orifice at time t. Then $\tau(t) = A_1(t)/A$ describes the relative size of the opening compared with the original area. It is this quantity τ that is given as a function of the time t. Typical closure descriptions may be $\tau(t) = (1 - t/T)^m$, where T is the closure time and m is a positive parameter which distinguishes between valves. Notice that $\tau(0) = 1$, (no closing has been effected) and $\tau(T) = 0$, (the valve is closed). For this problem we select $T = 90$ sec. and $0.5 < m < 1.5$.

Examining τ for various choices of m reveals that for $0.5 < m < 1$, the valve closes slower initially than it does near $t = T$. For $1 < m < 1.5$, the opposite is true. For $m = 1$, the closure is linear in t. You are to find the closure description τ (actually m) that produces the smallest maximum head at the valve, assuming an initial head of 551.0 feet.

You are also required to construct a computer program simulating the shock wave for 110 seconds after the valve begins to close. For various closure description, m is not a continuous parameter, but rather is incremented by steps of 0.1. Thus, you are to consider the eleven values $m = 0.5, 0.6, 0.7, \ldots, 1.5$.

As additional and useful information, you are to construct graphs of the head at the valve versus time for selected values of m, and determine the time at which the maximum head is reached. Furthermore, one might choose other closure descriptions than suggested above and simulate the effects at the valve.