TOPICS IN GEOMETRY: SHEAF THEORY
MATH 6490, SPRING 2009
HOMEWORK 8

Exercise 1. Let $i : A \hookrightarrow X$ be a closed embedding. Show that i_* is an exact functor. Conclude that for any sheaf $\mathcal{F}$ on A we have $H^q(A, \mathcal{F}) \simeq H^q(X, i_*\mathcal{F})$.

Exercise 2. Let $f : X \to Y$ be a continuous map. Show that f pulls back constant sheaves to constant sheaves, i.e., for any abelian group A, show that $f^*A_Y = A_X$. Where is this exercise necessary in our discussions on functoriality?

Exercise 3. Let $I = [-1, 1]$ be the closed one-dimensional unit disc, and let A be an abelian group. Show that $H^q(I, A) = 0$ for all $q \geq 1$. *(Hint: See exercises 18–21 in page 65 of Harder's book.)*

Some exercises in point set topology.

Exercise 4. Let X be a locally compact Hausdorff space. For any open set U and for any $x \in U$ show that there is an open set V which is relatively compact (that is $\overline{V}$ is compact) and $x \in V \subset \overline{V} \subset U$. (This can be rephrased as: the set of relatively compact neighbourhoods of any point forms a neighbourhood basis at that point.)

Exercise 5. Let X and Y be locally compact Hausdorff spaces. Let X^+ and Y^+ be their one-point compactifications (look this up from some book on topology) with ∞_X and ∞_Y being the *points at infinity* in X^+ and Y^+. Let $f : X \to Y$ be a continuous map. Show that f is proper if and only if f extends to a continuous map $f^+ : X^+ \to Y^+$ with $f^+(x) = f(x)$ for all $x \in X$ and $f(\infty_X) = \infty_Y$.

Exercise 6. Let X be a Hausdorff space which is exhausted by compact subsets, i.e., there is a sequence of compact subsets $K_1 \subset K_2 \subset \cdots \subset K_n \subset \cdots$ such that $X = \cup K_n$ and $K_n \subset \mathrm{Int}(K_{n+1})$. Show that X is paracompact. (Here $\mathrm{Int}(A)$ stands for the interior of the subset A.)

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