TOPICS IN GEOMETRY: SHEAF THEORY
MATH 6490, SPRING 2009
HOMEWORK 4

Exercise 1. Let A be an abelian group. Show that $\mathrm{Ext}^1_{\mathbb{Z}}(\mathbb{Z}/m\mathbb{Z}, A) \simeq A/mA$.

Exercise 2. Let A and B be two finite abelian groups with relatively prime orders. Show that $\mathrm{Ext}^i_{\mathbb{Z}}(A, B) = 0$ for all $i \geq 0$.

Exercise 3. Let A and B be two abelian groups. Show that $\mathrm{Ext}^i_{\mathbb{Z}}(A, B) = 0$ for all $i \geq 2$.

Exercise 4. Let k be a field and let $R = k[X]/(X^2)$. Then the category of R-modules is the same as the category of k-vector spaces V together with a k-linear endomorphism $\alpha : V \to V$ such that $\alpha^2 = 0$. (Think of the image of X in R acting on V via α.) If $\dim_k(V) = 1$ then $\alpha = 0$. Compute $\mathrm{Ext}^1_R(k, k)$.

Exercise 5. Compute $\mathrm{Tor}_1^{\mathbb{Z}}(\mathbb{Z}/m\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$.

Exercise 6. Let A and B be two abelian groups. Show that $\mathrm{Tor}_1^{\mathbb{Z}}(A, B)$ is a torsion group (meaning that every element has finite order).

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