Solutions.

CALCULUS-II, MATH 2153-006, 2-APR-2009
QUIZ-5

Evaluate the integral

$$\int \frac{\ln(1-t)}{t}\,dt$$

as a power series. What is the radius of convergence?

The function $\frac{1}{1-t}$ is represented by the "geometric" power series:-

$$\frac{1}{1-t} = 1 + t + t^2 + t^3 + \cdots + t^n + \cdots, \quad |t| < 1.$$

Integrate both sides (Note:- Term-by-term integration does not change radius of convergence)

$$\int \frac{1}{1-t}\,dt = C + t + \frac{t^2}{2} + \frac{t^3}{3} + \cdots + \frac{t^{n+1}}{n+1} \cdots, \quad |t| < 1.$$

$$\Rightarrow \quad -\ln(1-t) = C + t + \frac{t^2}{2} + \cdots + \frac{t^{n+1}}{n+1} + \cdots, \quad |t| < 1.$$

Put $t = 0$ to get $C = 0$

Hence

$$\ln(1-t) = -t - \frac{t^2}{2} - \frac{t^3}{3} - \cdots - \frac{t^{n+1}}{n+1} + \cdots, \quad |t| < 1.$$

$$\Rightarrow \quad \frac{\ln(1-t)}{t} = -1 - \frac{t}{2} - \frac{t^2}{3} - \cdots - \frac{t^n}{n+1} + \cdots, \quad |t| < 1$$

Integrate both sides :-

$$\int \frac{\ln(1-t)}{t} = C - t - \frac{t^2}{2^2} - \frac{t^3}{3^2} - \cdots - \frac{t^{n+1}}{(n+1)^2} - \cdots, \quad |t| < 1$$

or

$$\boxed{\int \frac{\ln(1-t)}{t}\,dt = C - \sum_{n=1}^{\infty} \frac{t^n}{n^2} \quad \text{for some constant } C. \quad |t| < 1}$$

Radius of convergence $= 1$.