SOLUTIONS.

**CALCULUS-II, MATH 2153-006, 26-FEB-2009**
**QUIZ-3**

Determine whether the sequence

$$a_n = \frac{(\ln n)^2}{n}$$

converges or diverges. If it converges, find its limit.

Let $f(x) = (\ln x)^2$, $g(x) = x$. Then $a_n = \frac{f(n)}{g(n)}$.

$\therefore \lim_{n\to\infty} a_n = \lim_{x\to\infty} \frac{f(x)}{g(x)}$.

Now $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (\ln x)^2 \longrightarrow \infty$

& $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} x \longrightarrow \infty$

$\therefore \infty/\infty$ is an indeterminate form.

Apply L'Hospital's theorem.

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{(\ln x)^2}{x} = \lim_{x\to\infty} \frac{2(\ln x)\cdot \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{2\ln(x)}{x}$$

Observe that again we have an $\infty/\infty$ situation & so apply L'Hospital once again :-

$$\lim_{x\to\infty} \frac{2\ln x}{x} = \lim_{x\to\infty} \frac{(2\ln x)'}{x'} = \lim_{x\to\infty} \frac{2/x}{1} = \lim_{x\to\infty} \frac{2}{x} = 0.$$

$$\lim_{n\to\infty} a_n = \lim_{x\to\infty} \frac{2(\ln x)^2}{x} = \lim_{x\to\infty} \frac{2\ln x}{x} = \lim_{x\to\infty} 2/x = 0$$

$\Rightarrow$ $\{a_n\}$ converges to 0.