

# ON THE CORRESPONDENCE OF REPRESENTATIONS BETWEEN $GL(n)$ AND DIVISION ALGEBRAS

JOSHUA LANSKY AND A. RAGHURAM

ABSTRACT. For a division algebra  $D$  over a  $p$ -adic field  $F$ , we prove that depth is preserved under the correspondence of discrete series representations of  $GL_n(F)$  and irreducible representations of  $D^*$  by proving that an explicit relation holds between depth and conductor for all such representations. We also show that this relation holds for many (possibly all) discrete series representations of  $GL_2(D)$ .

## 1. INTRODUCTION

Let  $F$  be a non-Archimedean local field. Let  $G$  be the group of  $F$ -points of a connected reductive algebraic group defined over  $F$ . Let  $(\pi, V)$  be an irreducible admissible representation of  $G$  on a complex vector space  $V$ . To such a  $\pi$ , Moy and Prasad ([12],[13]) have attached a rational number  $\rho(\pi)$  called the *depth* of  $\pi$  (see §2.2). The theory of (unrefined) minimal  $K$ -types for  $\pi$  depends crucially on  $\rho(\pi)$ . In addition, the notion of depth is important in harmonic analysis on  $G$ .

For the moment let  $G$  be the group of units of a central simple algebra over  $F$ , i.e.,  $G = GL_m(D)$  for a division algebra  $D$  over  $F$ . For any irreducible representation  $\pi$  of  $G$ , Godement and Jacquet [6] associate a local  $L$ -function and  $\varepsilon$ -factor to  $\pi$ . From this  $\varepsilon$ -factor comes an integer invariant  $c(\pi)$  called the *conductor* of  $\pi$ . The main result of this note is that a certain relation holds between depth  $\rho(\pi)$  and conductor  $c(\pi)$  if  $G = D^*$  or if  $G = GL_n(F)$  and  $\pi$  is essentially square integrable (see Theorem 3.1), namely

$$\rho(\pi) = \max \left\{ \frac{c(\pi) - n}{n}, 0 \right\}.$$

We also prove this for a lot of representations  $\pi$  of  $G = GL_2(D)$  (see § 4). We expect this relation to be true for any essentially square integrable representation of any  $GL_m(D)$ . It is however easy to see via examples that one cannot expect any explicit relation between depth and conductor to hold for *every* representation of  $G$ .

Our motivation for proving such a result relating the depth and conductor is as follows. Very generally speaking one may ask: “How functorial is depth?” More specifically given two such groups  $G_1$  and  $G_2$ , supposing there is a map at the level of dual groups  ${}^L G_1 \rightarrow {}^L G_2$ , the Langlands principle of functoriality predicts the existence of a map from the representations of  $G_1$  to those of  $G_2$ . The first question is how does depth behave under such a map? A deeper (no pun intended) question is how do the  $K$ -types behave under such a map? This

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question is not new and has been investigated under some instances of functoriality; see for example [3],[4].

One such instance is the correspondence of representations between  $G'$  and  $GL_n(F)$  where  $G'$  is any inner form of the latter group. Such a correspondence is known to exist by the work of Deligne, Kazhdan and Vignéras [5] and also Rogawski [16]. (We need to assume here that the characteristic of  $F$  is 0.) We shall henceforth abbreviate it as the DKVR correspondence and we now describe it in a little more detail.

Let  $D$  be a central division algebra over  $F$  of index  $n$ , i.e.,  $\dim_F(D) = n^2$ . Let  $G' = GL_m(D)$  and  $G = GL_{mn}(F)$ . The DKVR correspondence asserts that there is a unique bijection  $\pi' \leftrightarrow \pi$  between essentially square integrable representations  $\pi'$  of  $G'$  and similar representations  $\pi$  of  $G$  which is characterized by a certain character identity (see §2.1). Under this correspondence the conductor is preserved. The main theorem of this paper therefore implies that  $\rho(\pi) = \rho(\pi')$  if  $m = 1$  (see Theorem 3.1), i.e., in the case when  $G' = D^*$  and  $G = GL_n(F)$ . This equality is also shown to hold for a large class of representations if  $m = 2$ .

In the case of  $D^*$ , the relation between depth and conductor is proved using a result of Koch and Zink [10]. For  $GL_n(F)$  the relation follows in the supercuspidal case from theorems of Bushnell and Fröhlich [2] and Bushnell [1] which imply that the normalized level  $l(\pi)$  of  $\pi$  is equal to the above function of  $c(\pi)$ . This result was communicated to us by J.-K. Yu in the form

$$\rho(\pi) = \frac{\text{swan}(\phi(\pi))}{n}$$

where  $\text{swan}(\phi(\pi))$  is the Swan conductor of the Langlands parameter  $\phi(\pi)$  associated to  $\pi$ . If  $\text{artin}(\phi(\pi))$  denotes the Artin conductor then since  $\phi(\pi)$  is irreducible ( $\pi$  being supercuspidal) we have  $\text{swan}(\phi(\pi)) = \text{artin}(\phi(\pi)) - n$ . The desired relation between conductor and depth then follows from the fact that the Langlands correspondence preserves conductor, i.e.,  $c(\pi) = \text{artin}(\phi(\pi))$ . We generalize this relation to the essentially square integrable case. The proofs of these results are given in §3.

In §4, the result is proved for  $G' = GL_2(D)$  using a computation involving the Bruhat-Tits building and results of a previous work of the second author and Dipendra Prasad [14] where many (possibly all) supercuspidal representations of  $G'$  were constructed. We prove that depth is preserved when  $\pi'$  is one of these supercuspidals or if  $\pi'$  is an essentially square integrable non-supercuspidal representation of  $G'$ .

It should be noted that the results of Bushnell and Fröhlich ([1] and [2]) can be shown to imply that  $l(\pi) = \max\{(c(\pi) - n)/n, 0\}$  for supercuspidal  $\pi$  for many groups of the form  $GL_m(D)$ , including all those in the preceding paragraph. We would like to point out that our arguments for the case of  $D^*$  and in the last section dealing with  $GL_2(D)$  are completely independent of [1] and [2].

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## 2. PRELIMINARIES

**2.1. DKVR Correspondence.** Let  $F$  be a non-Archimedean local field of characteristic 0. Let the ring of integers of  $F$  be  $\mathcal{O}_F$ . Let  $\mathfrak{P}_F$  be the unique maximal ideal in  $\mathcal{O}_F$ . Let  $q$  be the cardinality of the residue field  $k_F$  of  $F$ . Let  $D$  be a central division algebra over  $F$  of index  $n$ , i.e.,  $\dim_F(D) = n^2$ . Let  $G' = GL_m(D)$  and let  $G = GL_{mn}(F)$ .

An irreducible representation of  $G$  or  $G'$  is said to be essentially square integrable if one (and hence every) matrix coefficient is up to a twist square integrable modulo the center.

The *DKVR correspondence* asserts that there is a unique bijection  $\pi' \leftrightarrow \pi$  between essentially square integrable representations  $\pi'$  of  $G'$  and essentially square integrable representations  $\pi$  of  $G$  such that for all regular elliptic  $\gamma$ ,

$$(-1)^m \chi_{\pi'}(\gamma) = (-1)^{mn} \chi_{\pi}(\gamma)$$

where  $\chi_{\pi'}$  and  $\chi_{\pi}$  are the characters of  $\pi'$  and  $\pi$  respectively. This bijection satisfies a number of properties (see p. 34-35 of [5]). For us the most important one is the equality of epsilon factors

$$(-1)^m \varepsilon(s, \pi', \psi) = (-1)^{mn} \varepsilon(s, \pi, \psi).$$

where  $\psi$  is a non-trivial additive character of  $F$  normalized such that the largest fractional ideal of  $F$  on which  $\psi$  is trivial is  $\mathcal{O}_F$ . These epsilon factors have the form  $\varepsilon(s, \Pi, \psi) = Aq^{-c(\Pi)s}$  for a non-negative integer  $c(\Pi)$  which is called the *conductor* of  $\Pi$ . In particular we get

$$c(\pi') = c(\pi).$$

Our proof that depths match now rests on the observation that for both  $\pi$  and  $\pi'$  the depth bears the same formal relation to the conductor.

**2.2. Depth.** In this section we review the definition of depth of a representation. Just for this section, let  $G$  be the  $F$ -points of a connected reductive algebraic group defined over  $F$ . Let  $\mathcal{B} = \mathcal{B}(G)$  be the Bruhat-Tits building attached to  $G$ . For any  $x \in \mathcal{B}$  let  $G_x$  be the parahoric subgroup of  $G$  associated to  $x$ . Moy and Prasad have defined a decreasing filtration  $\{G_{x,r}\}$  of  $G_x$  indexed by non-negative real number  $r$ . (See § 3 of [13].) Let  $G_{x,r^+} := \cup_{s>r} G_{x,s}$ . The *depth*  $\rho(\pi)$  of an irreducible admissible representation  $(\pi, V)$  of  $G$  is the smallest non-negative number such that the space  $V^{G_{x,r^+}}$  is nontrivial for some  $x \in \mathcal{B}$  (see [12]).

We will need the following result of Moy and Prasad which says that depth is invariant under parabolic induction. (See Theorem 5.2 in [13].)

**Proposition 2.1.** *Let  $P$  be a parabolic subgroup of  $G$  with Levi decomposition  $P = MN$ . Let  $\tau$  be an irreducible admissible representation of  $M$ . Let  $\pi$  be an irreducible constituent of the parabolic induction  $\text{Ind}_P^G(\tau)$  of  $\tau$  to a representation of  $G$ . Then*

$$\rho(\pi) = \rho(\tau).$$

For irreducible supercuspidal representations of  $GL_n(F)$  we need the following result of Bushnell and Fröhlich (see Theorem 3.3.8 of [2], Theorem 3 and especially paragraph (5.1) of [1]). We note that the terminology of [1] is different and the result was communicated to us in the form below by J.-K. Yu in a form using the Swan Conductor of the Langlands

parameter associated to  $\pi$ . We prefer to write it as stated below since it is in this form that the formula admits generalizations to other representations and other related groups.

**Proposition 2.2.** *Let  $\pi$  be an irreducible supercuspidal representation of  $GL_n(F)$ . The depth  $\rho(\pi)$  of  $\pi$  is related to the conductor of  $\pi$  by the formula*

$$\rho(\pi) = \frac{c(\pi) - n}{n}.$$

**Remark 2.3.** The purpose of this note is to show that this formula continues to hold with a minor modification for any essentially square integrable representation of either  $GL_n(F)$  or  $D^*$  (and also for many such representations of  $GL_2(D)$ ).

### 3. MAIN THEOREM

**Theorem 3.1.** *Let  $\pi$  be either an irreducible representation of  $D^*$  or an essentially square integrable representation of  $GL_n(F)$ . The conductor  $c(\pi)$  of  $\pi$  and the depth  $\rho(\pi)$  of  $\pi$  are related by*

$$\rho(\pi) = \max \left\{ \frac{c(\pi) - n}{n}, 0 \right\}.$$

**Proof :** We begin with the division algebra. Let  $\mathfrak{P}$  denote the maximal ideal in the maximal compact subring  $\mathcal{O}$  of  $D$ . Let  $\pi$  be an irreducible representation of  $D^*$ . We recall the definition of *level* of  $\pi$ . For any non-negative integer  $k$ , let  $U_k = 1 + \mathfrak{P}^k$ , with the convention that  $U_0 = \mathcal{O}^\times$  which is the group of units in  $\mathcal{O}$ . The level of  $\pi$  denoted  $\ell(\pi)$  is the least non-negative integer  $m$  such that  $\pi$  is trivial on  $U_m$ . A standard computation involving local factors (see [10]) relates the conductor to the level by the formula

$$c(\pi) = \ell(\pi) + n - 1.$$

It is an easy exercise to see that if  $x$  is any point in the building of  $D^*$  then for any non-negative real number  $r$  we have

$$D_{x,r}^* = 1 + \mathfrak{P}^{\lceil nr \rceil}.$$

From this and the definition of depth, it is clear that  $\rho(\pi) = \max\{(\ell(\pi) - 1)/n, 0\}$  from which it follows that

$$\rho(\pi) = \max \left\{ \frac{c(\pi) - n}{n}, 0 \right\}.$$

Now let  $\pi$  be an essentially square integrable representation of  $GL_n(F)$ . According to the classification of such representations by Bernstein and Zelevinskii (see p.369-370 of [11]),  $\pi$  is equivalent to the Langlands quotient  $Q(\tau)$  of the parabolic induction  $\text{Ind}_P^G(\tau)$  where  $P$  is a standard parabolic subgroup of  $GL_n(F)$  whose Levi subgroup is of the form  $GL_a(F) \times \cdots \times GL_a(F)$  and  $\sigma$  is a representation of  $P$  of the form  $\tau \otimes \tau|\det| \otimes \cdots \otimes \tau|\det|^{b-1}$  where  $\tau$  is a supercuspidal representation of  $GL_a(F)$ . (Note that  $ab = n$ .) By Proposition 2.1 we get

$$\rho(\pi) = \rho(\tau \otimes \cdots \otimes \tau(b-1)).$$

It is easily seen that  $\rho(\tau \otimes \cdots \otimes \tau(b-1)) = \rho(\tau)$ . Hence we have

$$\rho(\pi) = \rho(\tau).$$

We now consider two cases. First suppose that  $a = 1$ . Then  $\tau$  is just a character of  $F^*$ . We can view  $F^*$  as a special case of a division algebra of index 1 and so by the preceding discussion we have

$$\rho(\tau) = \max \left\{ \frac{c(\tau) - a}{a}, 0 \right\}.$$

Next we consider the case where  $a > 1$ . Now  $\tau$  is an irreducible supercuspidal representation of  $GL_a(F)$ . Then by Proposition 2.2 we have

$$\rho(\pi) = \frac{c(\tau) - a}{a} = \max \left\{ \frac{c(\tau) - a}{a}, 0 \right\}.$$

(For the last equality we have used that the conductor of a supercuspidal representation  $\tau$  of  $GL_a(F)$  is at least  $a$ .)

We now further analyze the conductor of  $\pi = Q(\tau)$ . For this we need the following formula describing the epsilon factor of such a  $\pi$  in terms of the local factors of  $\tau$  and its contragredient  $\tau^\vee$ . (See Equation (5), § 2.6 of [7].)

$$(3.2) \quad \varepsilon(s, \pi, \psi) = \prod_{i=0}^{b-1} \varepsilon(s + i, \tau, \psi) \prod_{i=0}^{b-2} \frac{L(-s - i, \tau^\vee)}{L(s + i, \tau)}.$$

We consider two cases again. First suppose that  $a = 1$  and  $\tau$  is unramified. This implies that  $\pi$  is an unramified twist of the Steinberg representation  $\text{St}_n$  of  $GL_n(F)$ . Hence the conductor of  $\pi$  is the conductor of  $\text{St}_n$ . (By the main theorem of [9].) An easy exercise using the formula above gives  $c(\text{St}_n) = n - 1$ . Also since  $\pi$  is a constituent of an unramified principal series representation we get, using [13], that  $\rho(\pi) = 0$ . Hence

$$\rho(\pi) = 0 = \max \left\{ \frac{c(\pi) - n}{n}, 0 \right\}.$$

We now consider the case where  $a > 1$  or where  $a = 1$  and  $\tau$  is ramified. It is well known that in this case we have  $L(s, \tau) = L(s, \tau^\vee) = 1$ . Using the above formula for the epsilon factor for  $\pi$  we get  $c(\pi) = c(\tau)b$ . Hence again we have

$$\rho(\pi) = \max \left\{ \frac{c(\tau) - a}{a}, 0 \right\} = \max \left\{ \frac{c(\pi) - n}{n}, 0 \right\}.$$

□

**Corollary 3.3.** *Under the correspondence between irreducible representations  $\pi'$  of  $D^*$  and essentially square integrable representations  $\pi$  of  $GL_n(F)$ , depths are preserved, i.e.,*

$$\rho(\pi') = \rho(\pi).$$

**Proof :** The corollary follows from Theorem 3.1 since  $c(\pi) = c(\pi')$ . □

#### 4. THE CASE OF $GL_2(D)$

For this section let  $G' = GL_2(D)$ . We begin by briefly recalling a construction of supercuspidal representations of  $G'$  by D. Prasad and the second author. (See § 5 in [14] and also § 3.3 of [15].) We will need to revisit the filtrations introduced there and recognize them as Moy-Prasad filtrations.

Let  $K(m) = 1 + \mathfrak{P}^m M_2(\mathcal{O})$  for  $m \geq 1$ . Let  $K(0) = GL_2(\mathcal{O})$  and let  $H_1$  be the normalizer in  $G'$  of  $K(0)$ . It is an easy exercise using § 3 of [13] to see that there is a point  $x$  in the Bruhat-Tits building  $\mathcal{B} = \mathcal{B}(GL_2(D))$  such that for any non-negative real number  $r$  we have

$$GL_2(D)_{x,r} = K(\lceil nr \rceil).$$

Now let  $I(0) = \begin{bmatrix} \mathcal{O}^\times & \mathcal{O} \\ \mathfrak{P} & \mathcal{O}^\times \end{bmatrix}$  denote the subgroup of  $GL_2(\mathcal{O})$  which is upper triangular modulo  $\mathfrak{P}$ . (The notation is self explanatory.) Let  $I(2m) = \begin{bmatrix} 1 + \mathfrak{P}^m & \mathfrak{P}^m \\ \mathfrak{P}^{m+1} & 1 + \mathfrak{P}^m \end{bmatrix}$  and  $I(2m+1) = \begin{bmatrix} 1 + \mathfrak{P}^{m+1} & \mathfrak{P}^m \\ \mathfrak{P}^{m+1} & 1 + \mathfrak{P}^{m+1} \end{bmatrix}$  for any  $m \geq 1$ . Let  $H_2$  be the normalizer in  $G'$  of  $I(0)$ . Using § 3 of [13] it can be seen that there is a point  $y \in \mathcal{B}$  such that for any non-negative real number we have

$$GL_2(D)_{y,r} = I(\lceil 2nr \rceil).$$

(Recall from Proposition 1.4 of [14] that  $H_1$  and  $H_2$  are up to conjugacy the two maximal open compact-mod-center subgroups of  $G'$ . We also would like to point out that in the notation of [14] we have  $H_1(m) = K(m)$  and  $H_2(m) = I(2m)$ .) Finally it can be seen that for any  $z \in \mathcal{B}$  and any non-negative real number  $t$ , the group  $G'_{z,t}$  can be conjugated to either  $G'_{x,r}$  or  $G'_{y,r}$  for some  $r$ . In other words it suffices to consider just these two filtrations corresponding to the points  $x$  and  $y$ .

Let  $H = H_1$  or  $H_2$ . Let  $m \geq 1$  and let  $(\sigma, W)$  be a very cuspidal representation of  $H$  of level  $m$ . (See Definition 5.1 of [14].) Let  $\pi = \text{ind}_H^{G'}(\sigma)$  be the compact induction of  $\sigma$  to a representation of  $G'$ . Proposition 5.1 of [14] says that such a  $\pi$  is an irreducible supercuspidal representation of  $G'$ . Also Proposition 5.2 of [14] computes the conductor of  $\pi$  and this is given by

$$c(\pi) = 2m + i - 1 + 2(n - 1)$$

where  $i$  is such that  $H = H_i$ .

Now comparing the definition of an unrefined minimal  $K$ -type of Moy and Prasad and the definition of a very cuspidal representation (at this point it is better to use the second author's thesis [15] rather than [14]) leads us to the following easy observations whose proof we omit.

- (1) For  $m \geq 1$ , if  $\sigma$  is a very cuspidal representation of  $H_1$  of level  $m$  then  $\sigma$  contains an unrefined minimal  $K$ -type of depth  $(m - 1)/n = (2m - 2)/2n$ .
- (2) For  $m \geq 1$ , if  $\sigma$  is a very cuspidal representation of  $H_2$  of level  $m$  then  $\sigma$  contains an unrefined minimal  $K$ -type of depth  $(2m - 1)/2n$ .

In particular if  $H = H_i$  and  $\pi = \text{ind}_H^{G'}(\sigma)$  we get

$$\rho(\pi) = \frac{2m + i - 3}{2n} = \frac{c(\pi) - 2n}{2n}.$$

In other words the required relation between depth and conductor holds for every supercuspidal representation constructed in [14].

We now consider essentially square integrable representations of  $G'$  which are not supercuspidal. (See Theorem B.2.b of [5].) They are obtained as follows. Let  $\tau$  be an irreducible representation of  $D^*$ . Then there is a unique unramified character  $\nu$  such that  $\tau \otimes \tau\nu$  as a representation of  $D^* \times D^*$  gives a reducible representation when parabolically induced to  $G'$ .

This induced representation has a unique irreducible quotient, which we denote  $Q(\tau)$ . This  $Q(\tau)$  is essentially square integrable and every essentially square integrable representation of  $G'$  which is not supercuspidal is equivalent to a  $Q(\tau)$  for a uniquely determined  $\tau$ .

By Proposition 2.1 and the division algebra part of the proof of our Theorem 3.1 we get

$$\rho(Q(\tau)) = \rho(\tau) = \max \left\{ \frac{c(\tau) - n}{n}, 0 \right\} = \max \left\{ \frac{2c(\tau) - 2n}{2n}, 0 \right\}.$$

As in § 3 we can similarly compute the conductor of  $Q(\tau)$  and get  $c(Q(\tau)) = 2c(\tau)$  if  $c(\tau) \geq n$  or that  $c(Q(\tau)) < 2n$ . (This can be seen using Equation (3.2) and Theorem B.2.b of [5].) This latter case happens when  $c(\tau) < n$  and hence  $\ell(\tau) = 0$ , i.e.,  $\tau$  is an unramified character of  $D^*$  which would give that  $\rho(Q(\tau)) = 0$ . Hence

$$\rho(Q(\tau)) = \max \left\{ \frac{c(Q(\tau)) - 2n}{2n}, 0 \right\}.$$

To summarize, we have proved the following proposition in this section.

**Proposition 4.1.** *Let  $\pi'$  be either a supercuspidal representation constructed in [14] or an essentially square integrable non-supercuspidal representation of  $GL_2(D)$ . Then*

$$\rho(\pi') = \max \left\{ \frac{c(\pi') - 2n}{2n}, 0 \right\}.$$

**Corollary 4.2.** *Let  $G' = GL_2(D)$  and let  $G = GL_{2n}(F)$ . Under the DKVR correspondence between essentially square integrable representations of  $G'$  and those of  $G$  we have*

$$\rho(\pi') = \rho(\pi)$$

*if  $\pi'$  is either a supercuspidal representation constructed in [14] or is an essentially square integrable non-supercuspidal representation of  $G'$ .*

We end this article by stating the following conjecture.

**Conjecture 4.3.** *Let  $A$  be a central simple algebra over  $F$  such that  $\dim_F(A) = n^2$ . Let  $G = A^\times$ . Let  $\pi$  be an essentially square integrable representation of  $G$ . Then the relation between the depth  $\rho(\pi)$  and the conductor  $c(\pi)$  of  $\pi$  is given by*

$$\rho(\pi) = \max \left\{ \frac{c(\pi) - n}{n}, 0 \right\}.$$

As in the proof of Theorem 3.1 using Equation (3.2) and Theorem B.2.b of [5] it suffices to prove this conjecture for supercuspidal representations of  $G$ . It is tempting to speculate that a similar equality holds for any reductive  $G$  for which there is a theory of epsilon factors (which would give us the conductor  $c(\pi)$ ) with  $n$  replaced by the absolute rank of  $G$ .

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*Address:*

J. Lansky, Mathematics Department, 380 Olin Science Building, Bucknell University, Lewisburg, PA 17837, USA. E-mail : [jlansky@bucknell.edu](mailto:jlansky@bucknell.edu)

A. Raghuram, School of Mathematics, Tata Institute of Fundamental Research, Dr. Homi Bhabha Road, Colaba, Mumbai - 400005, INDIA. E-mail : [raghuram@math.tifr.res.in](mailto:raghuram@math.tifr.res.in)